Catenary Solar Cookers

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Abstract:

The purpose of this investigation was to discover if catenary solar cookers can be used as a cheap alternative to parabolic or circular solar cookers. In my research, I used Geometry Expressions, a constraint-based geometry system combined with a simple computer algebra system, and Maple, an advanced computer algebra system. I modeled a catenary solar cooker in Geometry Expressions and used Maple to calculate the percentage of light rays that reflected into a specified radius. The results proved that a parabolic solar cooker receives more total rays of light per day than a catenary solar cooker (only looking at when 50% or more of the reflected light is hitting the target). However, the results only account for cases when the sun is directly overhead (at the Equator). In order to account for cases where the sun is not overhead, I would have to model a tilted catenary solar cooker, which I may do in a subsequent investigation. In conclusion, the catenary solar cooker is a cheap and relatively effective design for use on the Equator, and has potential for use at other latitudes.

Introduction:

Solar cookers are gaining popularity around the world because of their ability to cook or sterilize food without using any fuel. Dwindling natural resources make solar cookers increasingly applicable to developing and poverty-stricken countries. Many people, particularly women, are forced to travel tens of miles to gather firewood or fuel, exposing them to rape, kidnap, or murder. Solar cookers significantly reduce this risk by providing a fuel-free way to cook meals. Another advantage of solar cookers is that they don't generate smoke, so the likelihood of a house fire is reduced as well as the chance of inhaling harmful pollutants. With all of these benefits in mind, it becomes clear that solar cookers should become more available for those in need.

Fig. 1: A man admires his parabolic solar cooker.

The aim of this investigation is to discover whether a catenary solar cooker is as effective as a parabolic solar cooker (**Figure 1**). A catenary is defined by Merriam Webster as the curve assumed by a cord of uniform density and cross section that is perfectly flexible but not capable of being stretched and that hangs freely from two fixed points. For example, if a chain was suspended from two posts, it would form a catenary (**Figure 2**).

Fig. 2: A catenary formed by a hanging chain.

Fig. 3: The Gateway Arch is a flattened catenary.

The catenary has historically been used in architecture (**Figure 3**) and in some kilns because of the way that they distribute their weight. All catenaries are similar. Mathematically, the catenary is defined by the equation $y=a*\cosh(x/a)$. I chose to investigate the catenary because it, if designed correctly, could potentially be manufactured for a significantly lower cost than a circular or parabolic solar cooker, therefore making it more readily available for people in need.

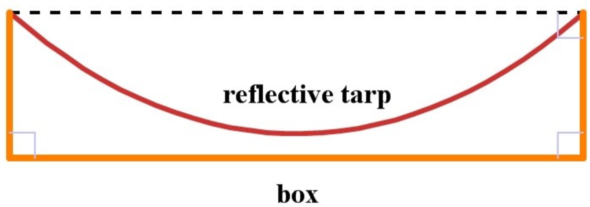

Fig. 4: My initial catenary solar cooker design.

Figure 4 shows my initial conceptual design. It consists of two main parts. One part is the box. This would be made out of a cheap, sturdy material like wood. The other part is the reflective tarp. This could be made out of Mylar or a light, heat-resistant tent fabric coated in reflective paint. My design presents a few questions. How much should the tarp sag? And most importantly, Is this design actually effective for its intended uses?

Procedure:

As an intern at Saltire Software, I had the opportunity to use Geometry Expressions to solve various math problems. Geometry Expressions is a constraint-based geometry system combined with a simple computer algebra system that was designed by Saltire Software. In order to answer the questions that my design posed, I again looked to Geometry Expressions.

I started by graphing $a*\cosh(x/a)$ in Geometry Expressions. Then I drew a line through a point t along the catenary to represent the sun's rays. I constrained this line to be angle $(\pi*\theta/180)$ with the x-axis. Next I reflected that line over the tangent to the catenary at that point to represent the reflected ray. After that, I drew the locus of the reflected ray as t varied from -1.0 to 1.0. To represent the diameter of the solar cooker, I connected points h*a and $-h*a$ along the catenary. This generalizes the diameter for any value of a so that I need only to change h. Finally, to create the cooking pot, I drew a circle with its center at the intersection of line $h*a$ and the y-axis. I constrained the radius of the circle to be $k*h*a$ and locked k to be an arbitrary value (in this case, k=0.2). This allowed me to preserve the ratio between the radius of the pot and the diameter of the solar cooker.

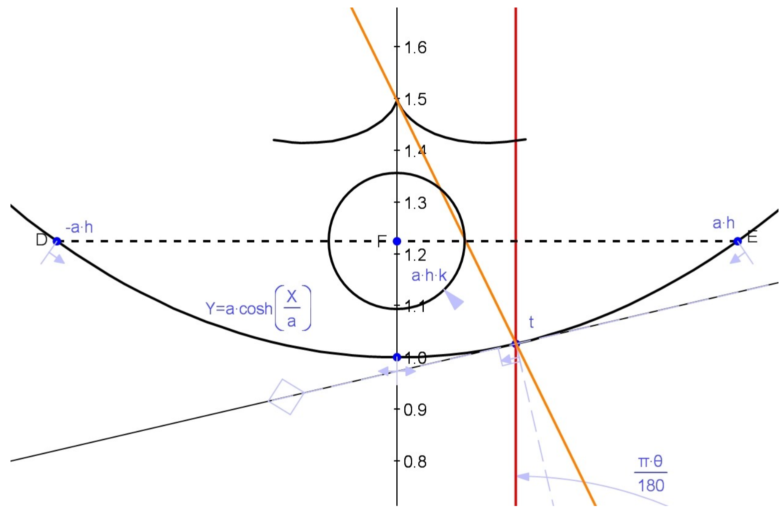

Fig. 5: The solar cooker modeled in Geometry Expressions.

Observations:

In this model, *a* only changes the size of the catenary. Since all catenaries are similar, changing *a* can be thought of as zooming in or out. The radius of the pot is represented by *k*. The bigger the value of *k*, the larger the cooking pot. The angle of the sun is represented by θ. The sag of the reflective tarp is represented by *h*. As *h* increases, the diameter of the solar cooker moves "higher up" the catenary, and the radius of the pot "increases". In reality, the diameter of the cooker and the radius of the pot are not really changing with *h*. The only difference is the distance between the center of the cooking pot and the bottom of the reflective tarp. This is important because we want to limit the variables used. Imagine that we have built the solar cooker with a width of one meter, and we have a pot with a radius of 10 centimeters. The only variable we want to change is how much the reflective fabric underneath the pot sags, because we have already made the box. In this case, $2*h*a$ is equal to one meter, and *k* is set to equal .2. By changing *h* in the diagram we are only changing the length of the fabric and therefore the amount of sag. So, in order to answer my question, "How much sag?", I needed to find the best value of *h*.

Optimizing the tarp's sag:

To determine the optimal amount of sag (*h*), I looked to Maple, an advanced computer algebra system. I utilized the raytrace function, which essentially projects a specified number of evenly spaced rays at a curve and returns the percentage of reflected rays that "hit" within a given radius. Then I graphed how the percentage of hits, or percent efficiency, changed with θ for different values of *h*. Considering that people using a solar cooker of this sort wouldn't be able to cook much of anything under 50% efficiency, I looked only at values of efficiency over 50%.

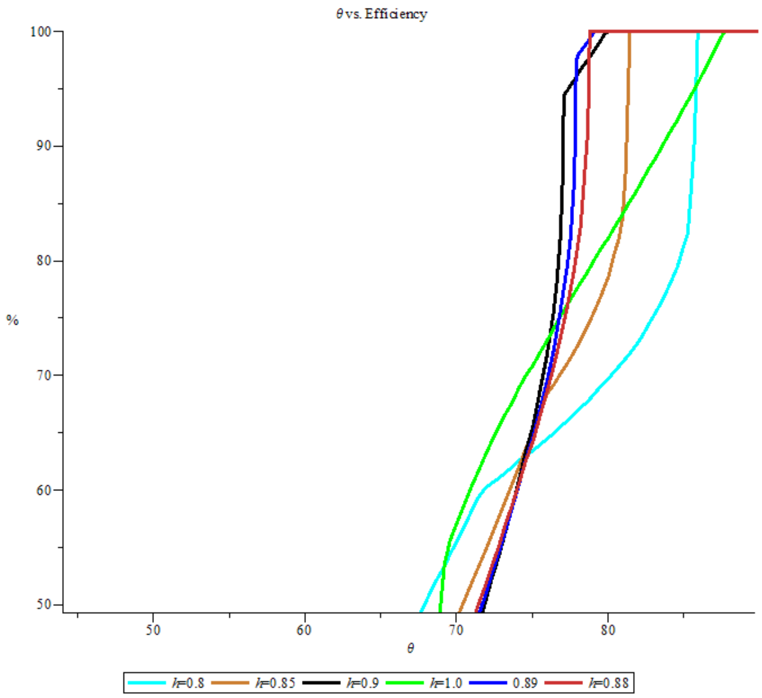

Fig. 6: The *theta* vs. efficiency graph showing only efficiencies above 50%.

This graph allowed me to closely examine each value of h. It appeared to me as though the optimal range was between h=0.88 and h=0.9 because they seemed to have the largest area between the other curves. When I integrated with respect to θ for values of efficiency over 50%, I found that h=0.9 was the optimal value.

Catenary vs. Parabola:

Now that I knew the best value of h, I was able to compare its effectiveness with that of the parabola that most closely matched the catenary. I first guessed using geometry that the closest parabola to the catenary was one with a vertex at a along the y-axis (the vertex of the catenary), and a focus at 3*a/2 (the cusp of the locus of the reflected rays along the catenary).

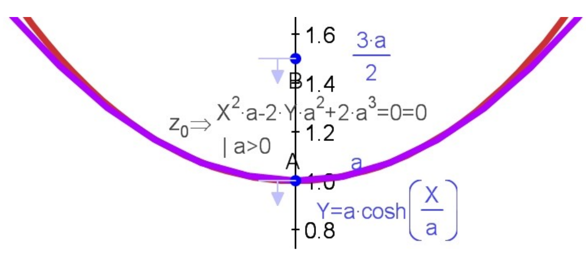

Fig. 7: A catenary (red) and my guess of the closest approximating parabola (purple).

However, a guess was not sufficient so I used Taylor series to find the closest approximating curve.

$$\cosh x = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots \text{ for all } x$$

Fig. 8: The Taylor series of the hyperbolic cosine.

I used Maple to calculate the Taylor series when x was 0..3 in order to find the quadratic approximating function. The result when I compared it to my guess was that they were the same.

$$f(x) := a \cdot \cosh\left(\frac{x}{a}\right);$$

$$x \to a \cosh\left(\frac{x}{a}\right)$$

$$taylor(f(x), x = 0, 3);$$

$$a + \frac{1}{2a} x^2 + O(x^4)$$

$$catenarytaylorseries := a + \frac{1}{2a} x^2;$$

$$a + \frac{1}{2} \frac{x^2}{a}$$

$$((a^*(x)\hat{}(2)) + ((a)\hat{}(2)^*y^*(-2)) + ((a)\hat{}(3)^*2)) = 0$$

$$a x^2 - 2 a^2 y + 2 a^3 = 0$$

$$solve(\%, y);$$

$$\frac{1}{2} \frac{2a^2 + x^2}{a}$$

$$myguess := \frac{1}{2} \frac{2a^2 + x^2}{a};$$

$$\frac{1}{2} \frac{2a^2 + x^2}{a}$$

$$\frac{catenarytaylorseries}{myguess};$$

$$\frac{2\left(a + \frac{1}{2} \frac{x^2}{a}\right) a}{2a^2 + x^2}$$

$$simplify(\%);$$

$$1$$

Fig. 9: The Taylor series of the catenary is the same as my guess.

I graphed the raytrace of the best catenary and its parabola given by the Taylor series:

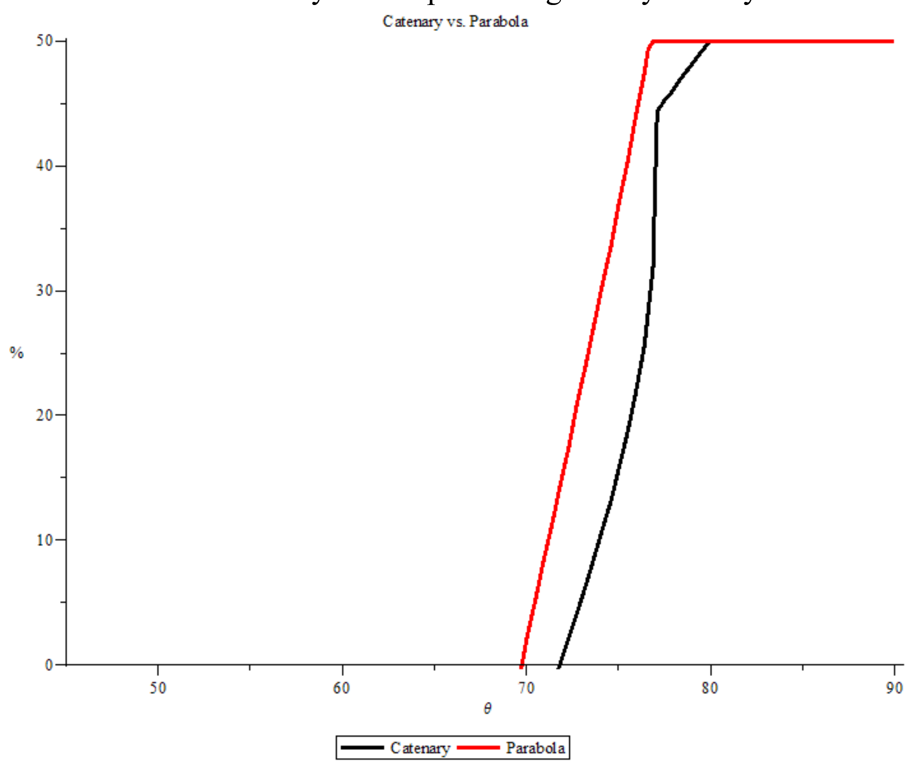

Fig. 10: The *theta* vs. efficiency graph displaying a catenary and its matching parabola.

It appeared that the parabolic solar cooker was more effective than the catenary solar cooker, and when

I integrated with respect to θ, the parabola indeed got more total sun for values of efficiency over 50% than the catenary.

Conclusion:

When h=0.9, the catenary solar cooker reflects the maximum amount of light rays onto a target. However, it is not as effective as the parabola that approximates it. This does not mean that the idea of a catenary solar cooker should be discarded. The fact that the catenary solar cooker is probably much cheaper to manufacture than a parabolic solar cooker makes it a strong competitor. This cheap yet relatively effective design could be used to fight hunger, murder, deforestation, desertification, and global warming. However, the results assume that the sun is directly overhead (locations along the Equator). For an arbitrary latitude (other than the Equator), the catenary solar cooker would have to be tilted, which would probably change the efficiency of the solar cooker. I trust that someone will carry out a subsequent investigation into the effectiveness of a tilted catenary solar cooker.

Bibliography

Fig. 1: Cement solar cooker [Online Image] Available **http://ghni.info/Portals/1/Afghanistan/cement%20solar%20cooker.jpg**, August 7, 2010.

Fig. 2: 400px-Catenary3 [Online Image] Available **http://mathforum.org/mathimages/imgUpload/thumb/Catenary3.jpg/400px-Catenary3.jpg**, August 7, 2010

Fig. 3: Gateway_Arch_10-5-05_071 [Online Image] Available **http://www.bvh.com/project_image/image/77/normal/Gateway_Arch_10-5-05_071.jpg**, August 18, 2010