What Does Geometry Expressions Calculate

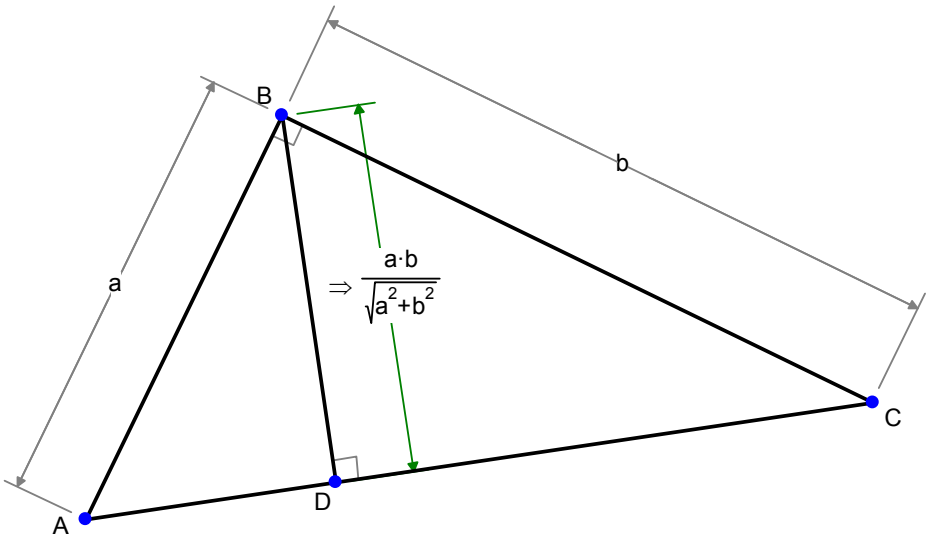
Constraints / Constructions

In order to work effectively with Geometry Expressions, it is useful to understand how its geometry model works.

Typical interactive geometry software has a construction based model. The user, specifies input geometrical entities, and specifies a set of constructions in order to define output entities.

For defining Symbolic Geometry models, it is more convenient to have a constraint based approach, where the user specifies the model by attaching symbolic constraints to a sketch of the geometry.

In the example below, AB is constrained to be length a, BC is constrained to be length b, AB is constrained to be perpendicular to BC, BD is constrained to be perpendicular to AC, and D is constrained to lie on AC.



Given this input, Geometry Expressions, internally creates a construction sequence for the drawing.

A construction sequence for the above drawing might be:

1. Place point A at an arbitrary location.
2. Put point B distance a from A in an arbitrary direction.
3. Create a line perpendicular to AB through B.
4. Put C on this line distance b from B
5. Find a line perpendicular to AC through B
6. Put D at the intersection of this line and AC

Once it has determined a construction sequence, Geometry Expressions will execute the constructions to create a symbolic model of the geometry. It will also use sample numerical values for any variables present in the definition in order to create a drawing which conforms to the constraints.

Which leads to a set of questions:

What is the effect of the arbitrary location and arbitrary direction of steps 1 and 2?

Geometry Expressions creates variables and sets them as the coordinates of A. By default, it will use the coordinates (u_0,v_0). It then creates another new variable and uses it as the slope of line AB. By default, it will use θ_0.

In step 4, how does Geometry Expressions decide which of two points distance b from B to use?

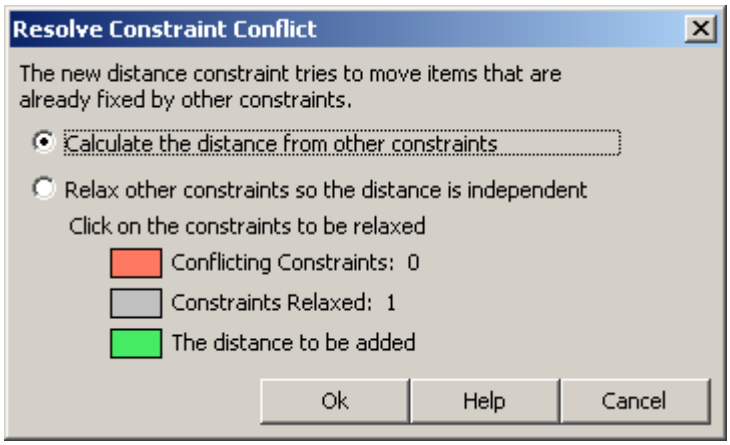
It uses the sketch to determine whether to use the point to the left of the line AB or the point to the right of the line AB.

How does Geometry Expressions come up with sample values for a and b?

In this case, it is easy, it simply measures the distance AB from the sketch and uses that as the sample value for **a**. It measures the distance BC from the sketch and uses that as the sample value of **b**. In other cases, when a variable is used in more than one place, or where it is involved in a complicated expression, Geometry Expressions does some numerical solving to come up with good sample values. However, whatever the values it finds, the user can always set his own values in the Variables Panel.

What happens if Geometry Expressions cannot find a construction sequence?

It gives you this dialog box:



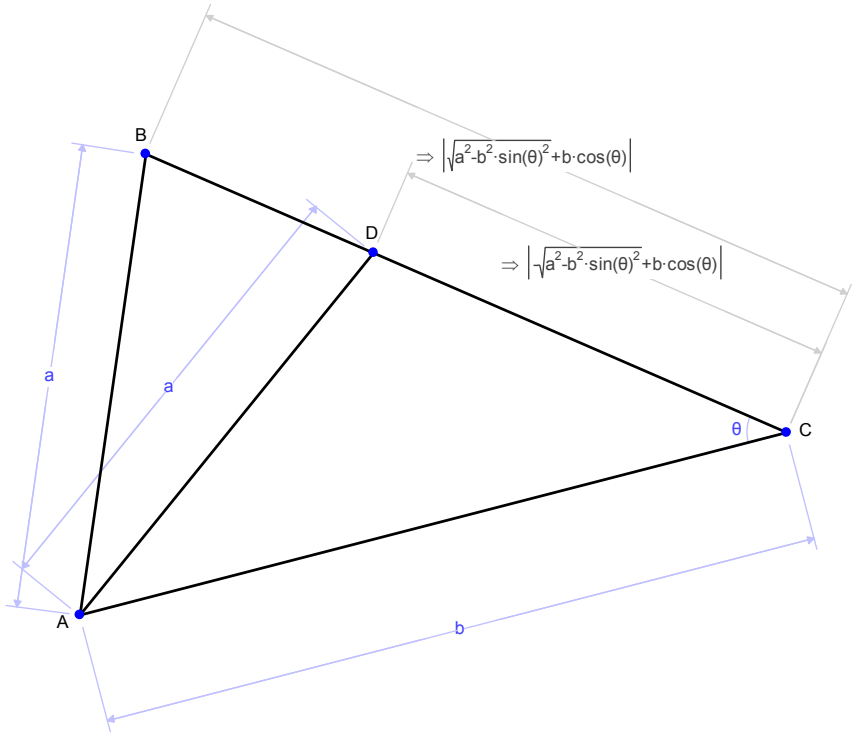
Which lets you relax other constraints, or cancel the addition of the constraint you have just added.

As Geometry Expressions is looking for a construction sequence, if you think in terms of a construction sequence when you are specifying your constraints, it can help explain the occasionally irritating appearance of this dialog.

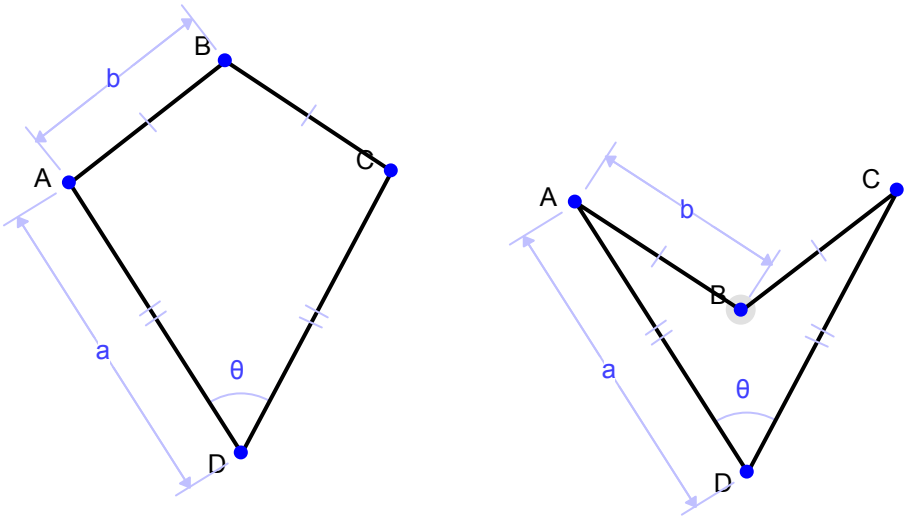
Resolving Geometrical Ambiguity

When we are taught geometry we learn that two sides and an included angle uniquely defines a triangle, but two sides and the non-included angle do not. In the case where the user has specified a triangle using two sides and the non-included angle, what is a conscientious symbolic geometry program to do? It can be pedantic and refuse to give a solution. Or it can be verbose and give both solutions. Or it can attempt to decide which solution the user intends and give him that one.

Geometry Expressions takes the latter approach, using the sketch as a guide. So in the following picture, triangles ABC and ADC both have sides length a and b and non-included angle θ. They are distinct because in the drawing ADC is obtuse while ABC is acute. Geometry Expressions uses the acuteness or obtuseness of this angle to discriminate which triangle is intended



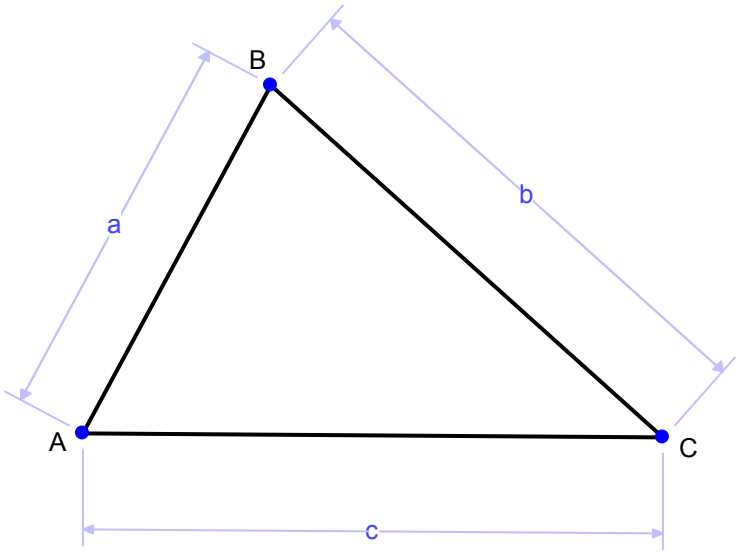
As another example, if we draw a kite with the following constraints, there are two possible solutions, one convex the other not. Again, Geometry Expressions makes the choice of which geometry is intended from the sketch.



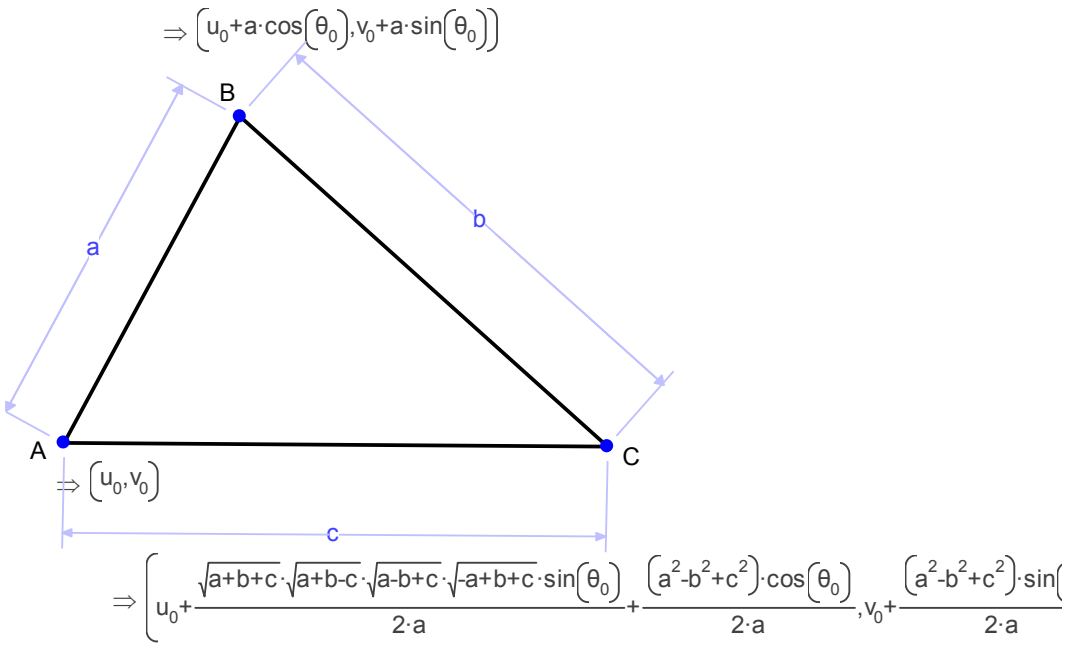
System Added Constraints

In Geometry Expressions, you do not need to fully constrain your model. Any parameters you have left unconstrained are filled in by the geometry engine.

For example, the following drawing is constrained only modulo a rigid transformation (typical for many geometry problems).



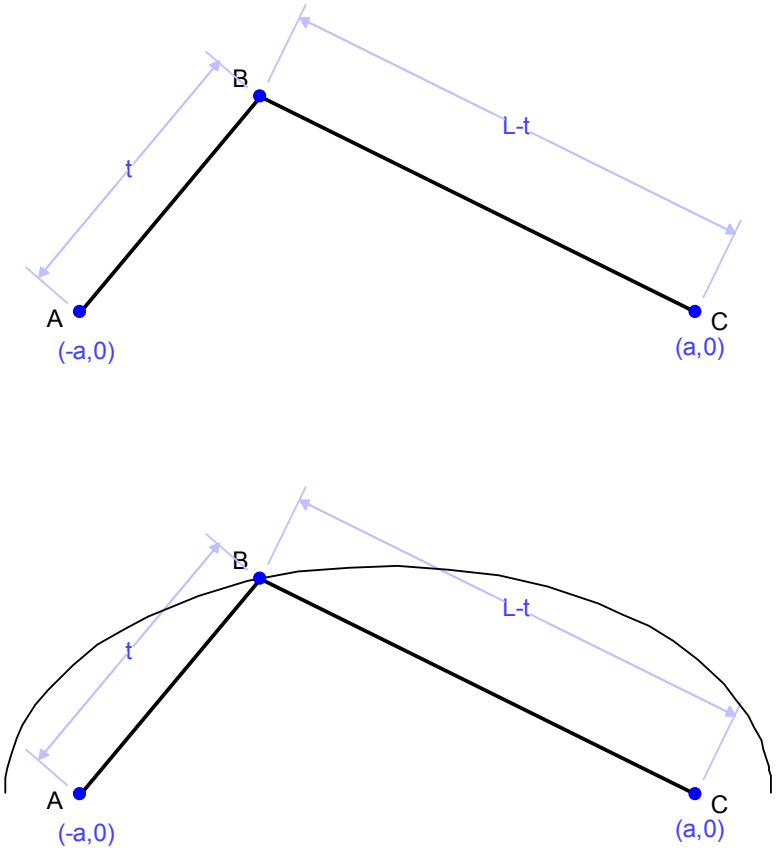
So if you ask for the coordinates of any of the points, they will be given in terms of system-added variables:



Clearly the system has added variables for the location of point A, and for the direction of line AB.

Locus & Animation

Both the construction of the locus and envelope curves, and the animation of the diagram in Geometry Expressions can be defined in terms of any variable. For example in the model below, we can create a locus over values of the variable t (other variables will be kept constant).



Users of numeric interactive geometry systems may be familiar with the concept of animation based on points animated along line segments or curves. This type of animation can be conveniently modeled in **Geometry Expressions** using the ***point proportional along a curve*** constraint along with parameter based animation.

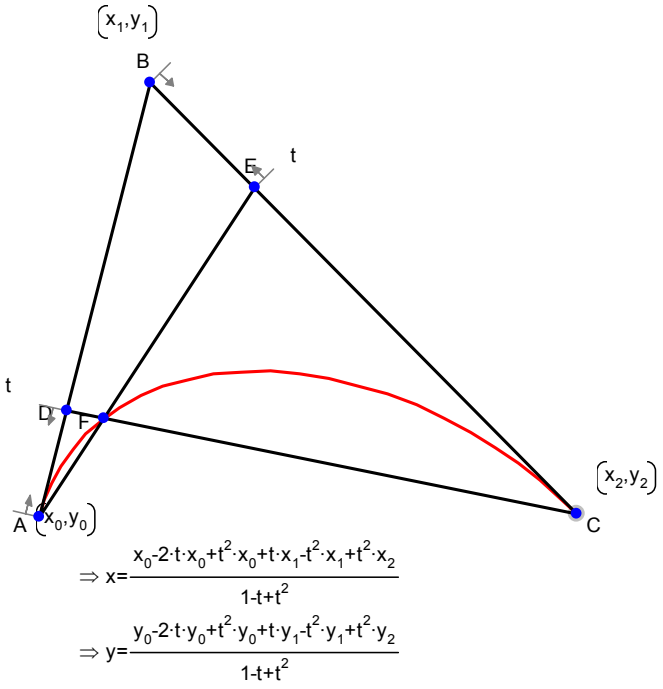
A point proportion t along a curve is defined variously for different types of curves as follows::

For a line segment AB, it defines the point (1-t) A + tB

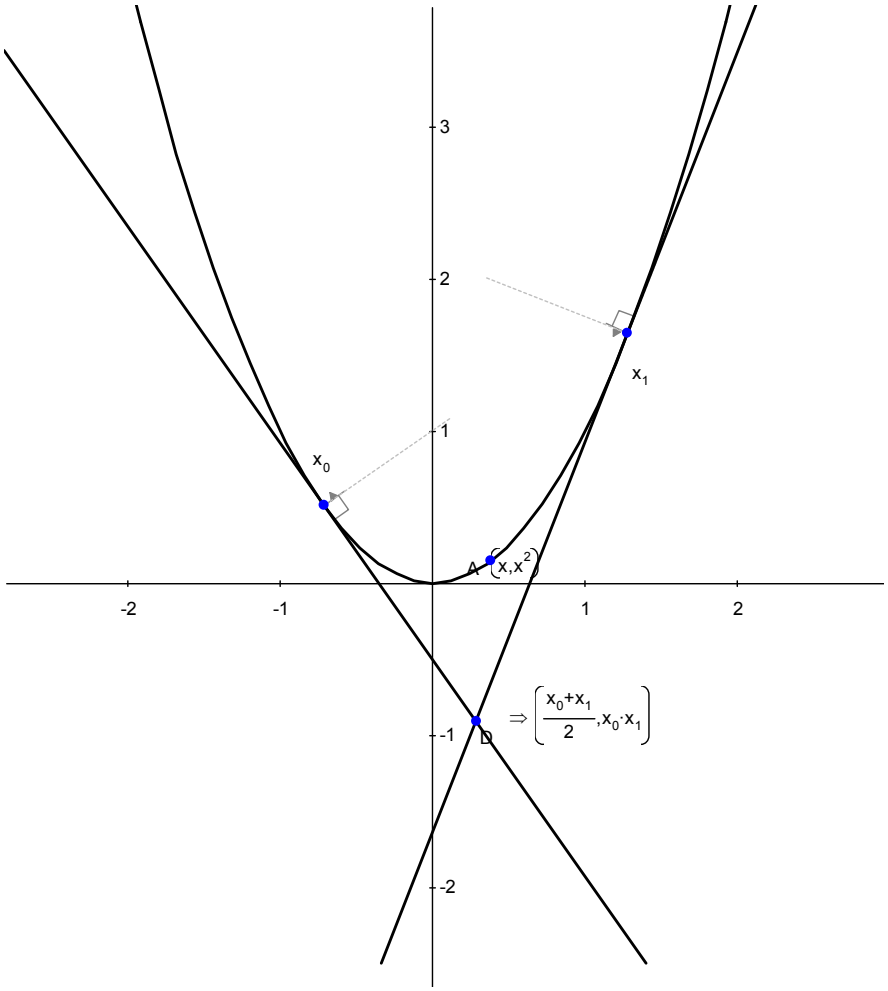
For a circle it defines the point on the circle which subtends angle t at the center.

For a locus or envelope, it defines the point at parameter value t.

For example, in the following diagram, D is defined proportion t along AB, and E is defined proportion t along BC. The curve is the locus of F as t varies between 0 and 1.



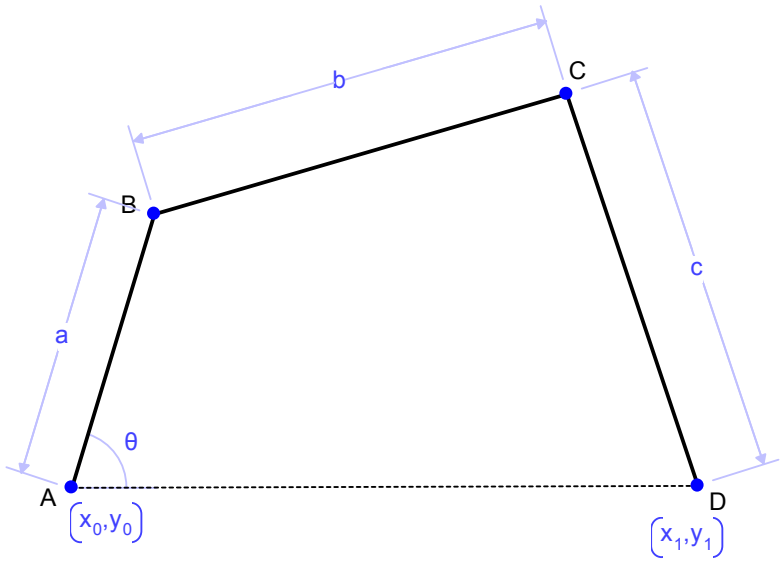
In the following example, the curve is the locus of the point (x,x^2). Tangents are created at points with parameter values x_0 and x_1 on this curve.



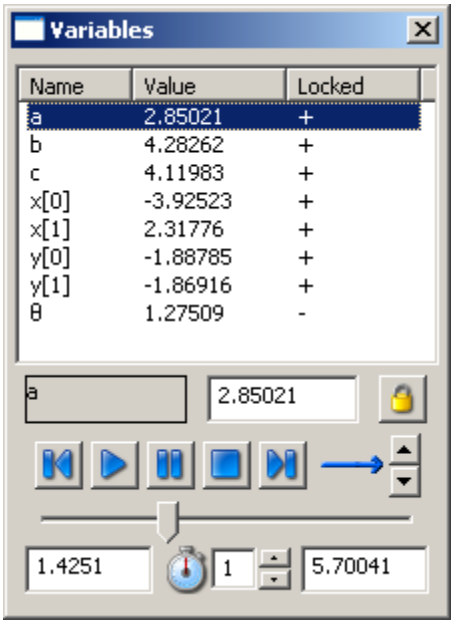
Locking & Dragging

By default, when you drag points in a Geometry Expressions model, it will adjust the numerical sample values used in the various parameters of the model to accommodate the drag, as best it can.

For example, in the model of a 4 bar linkage below, dragging point B will cause lengths a and b and angle θ to be adjusted appropriately.



However, you may want the drag to act as if the members AB and BC were rigid, and only angle θ adjustable. To do this you can lock parameters for dragging:



The value of a, for example, can still be set from the Variables panel, but it will not change when the model is dragged.