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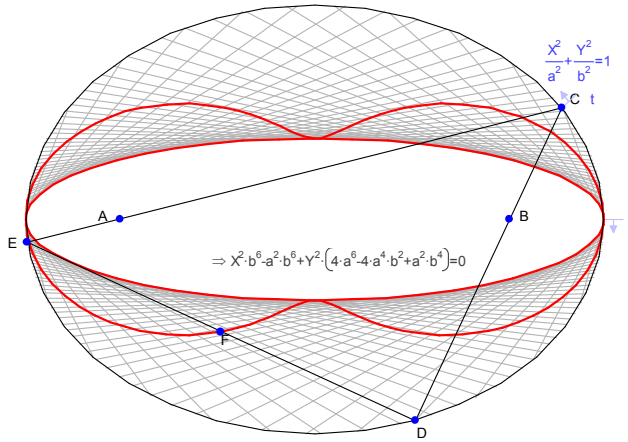
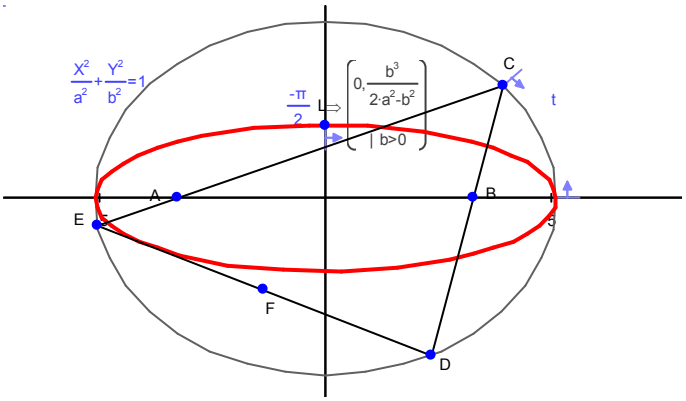
Given a point on the conic, we create the line joining the opposite ends of the two focal chords. We examine its envelope, as well as the locus of its center. Hint: turn off **Use Assumptions** in the **Math Properties** dialog.



$$\Rightarrow -Y^2\cdot a^4\cdot b^6+Y^4\cdot\left(4\cdot a^8-4\cdot a^6\cdot b^2+a^4\cdot b^4\right)+X^2\cdot Y^2\cdot\left(8\cdot a^6\cdot b^2-8\cdot a^4\cdot b^4+2\cdot a^2\cdot b^6\right)+X^4\cdot\left(4\cdot a^4\cdot b^4-4\cdot a^2\cdot b^6+b^8\right)+X^2\cdot\left(-4\cdot a^6\cdot b^4+4\cdot a^4\cdot b^6-a^2\cdot b^8\right)=0$$

The envelope is clearly an ellipse, from the equation, with width a, what is its height? Hint: place a point on the ellipse and make it $\frac{-\pi}{2}$ **Point proportional along curve**.