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Foreword

When we say “crop circles”, most people think of some sort of alien conspiracy theory associated with the bizarre, inexplicable, and sometimes very intricate patterns that mysteriously appear in fields around the world. This is not what we will be investigating here, though the geometric patterns associated with some of these designs are quite interesting. If you have ever flown across the United States and looked out of the window of the airplane, you may have noticed vast areas of the Midwest and West neatly dotted with circular patches of green. These patterns are a result of a widely popular method of farming that uses Center Pivot Irrigation. Long stretches of wheeled scaffolding, sometimes up to a mile in length, rotate around a fixed point that is located at a water source. These immense structures serve as a giant radius that results in a circular area capable of sustaining crops.

The typical student complaint about secondary mathematics is that it appears to have no practical application. The design and implementation of these systems provides us with an excellent example to see mathematics in action. As a farmer, it is very important to be able to effectively maximize your most important resources, in this case, land and water. Some of the problems that might need to be addressed are:

How can you irrigate the largest possible portion of your available land?

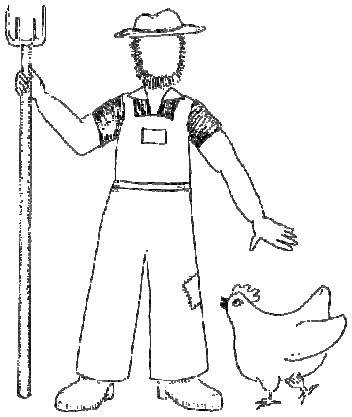
How can you minimize the costs involved?

How can you work around the existing features of your land?

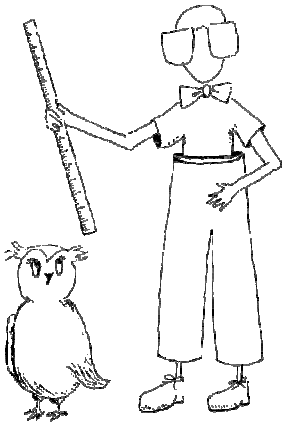
The proper design and implementation of these systems can and does involve precise mathematical calculations. New technology such as GPS, have made it even more possible to be very efficient. In a classroom setting, technology also makes it possible to investigate the rich and interesting mathematics that arises from looking at what some farmers have done and what they might consider doing. The lessons in this book utilize images from Google Earth, and a lighthearted scenario of a discussion between a farmer and a mathematician to introduce important mathematical concepts that are appropriate and adaptable for students in a range of courses from Pre-Algebra, Algebra, and Geometry, up through and including Calculus.

The process of mathematical modeling is both critical to using mathematics in the world around us, as well as developing an understanding and appreciation for the true utility of mathematics. A mathematician or engineer would first be given a problem to solve. They would also be given **constraints**, or a set of rules and conditions that must be followed. From that, the scientist would attempt to create a mathematical representation of the problem. Often, real problems are much too complex and it is necessary to constrain, or simplify them further. This is very similar to the approach that a student would take to gain insight into a problem by solving a simpler, related one. *Geometry Expressions* has been written to mirror this process. It is a ***constraint-based*** geometric and algebraic modeling program that allows the student to investigate problems from both a numeric and symbolic representation.

Lesson One



I have a square field that is approximately 1 mile in length on each side. I would like to use a single pivot point to irrigate as much land as possible. What percentage of my land will be effectively watered using this method?



Well, we intend to fill it with a circular irrigation area that touches each edge of your property without crossing. After all, we don't want to give up any of your water to a neighbor, or worse, waste it on the road! In mathematics, we refer to this as an **inscribed** circle and it is a very common situation that comes up in the study of Geometry. I will put my associates on it right away!

Quick Questions

1. What is the area of the square field?

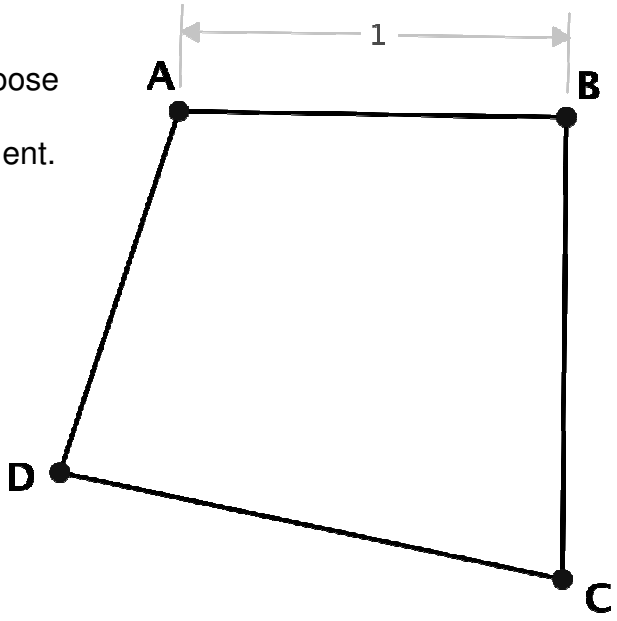
2. How would you describe where the pivot point for the irrigation system should be placed geometrically?

3. List the necessary geometric constraints of a square.

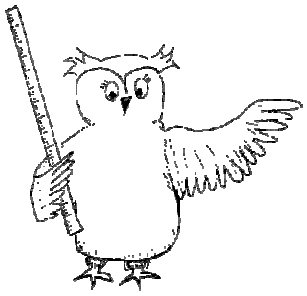
Informative Instructions

1. Launch *Geometry Expressions*. Choose **FILE-NEW** to open a new *Geometry Expressions* (*GX* to it's friends!) document. Save it as ***lesson1.gx***.

2. Choose **DRAW-Line Segment** to construct a quadrilateral ABCD. Don't waste time trying to make it square as we will do this next!

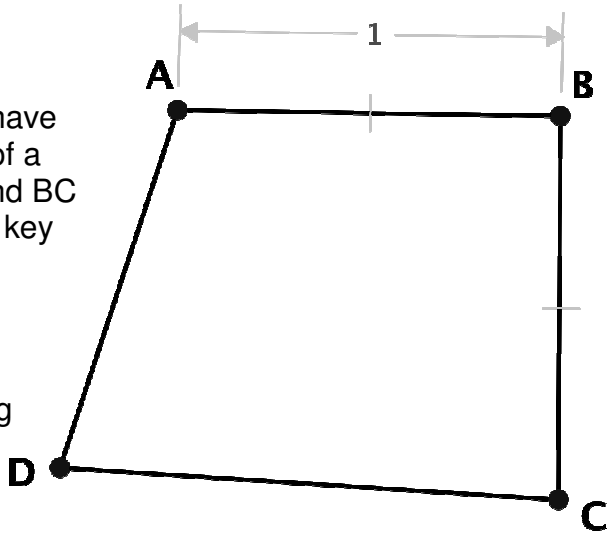


3. Constrain the length of one side to be one unit. Select **CONSTRAIN-DISTANCE/LENGTH** and then type 1.

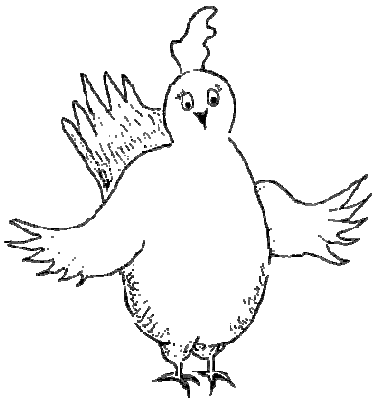


***Like most good software, GX allows you to access commands using icons or menus. If the Constrain Toolbox is visible, the icon for Distance/Length is . From now on in the text, we will use both the icons and the menu commands.

4. One of the properties you should have identified earlier is that all four sides of a square are equal. Select sides AB and BC of your square (hold down the **SHIFT** key while selecting the second side), and choose **CONSTRAIN-CONGRUENT** to force them to be the same length. Notice in the resulting drawing that they have small congruence symbols.



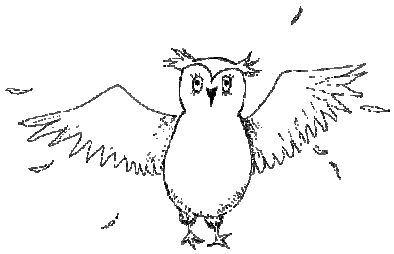
5. Repeat this procedure by constraining the remaining two sides to be congruent to the original (or a side that has already been constrained).



Depending on your initial drawing, your quadrilateral may look like a square, but it's not quite!! Compare yours with some of the other students around you. What is the most accurate name for the quadrilateral we have created?

6. We need to also constrain the angles to be right. Select two adjacent sides and choose **CONSTRAIN-PERPENDICULAR ( ).**

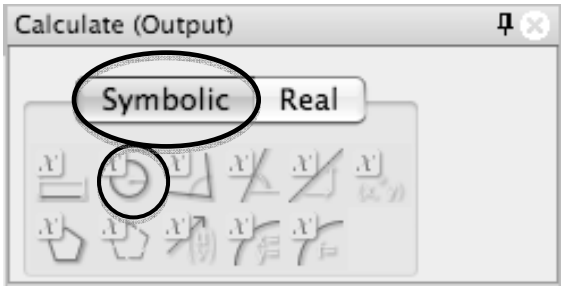
***Before completing the next step, GX is set, by default, to measure angles using radian measure. If you don't know what that is, don't panic because we will change it to degrees! Choose PREFERENCES-MATH PREFERENCES, and at the very bottom, find ANGLE and change it to degrees. Until you change it back, all angle measures will be displayed in degrees.



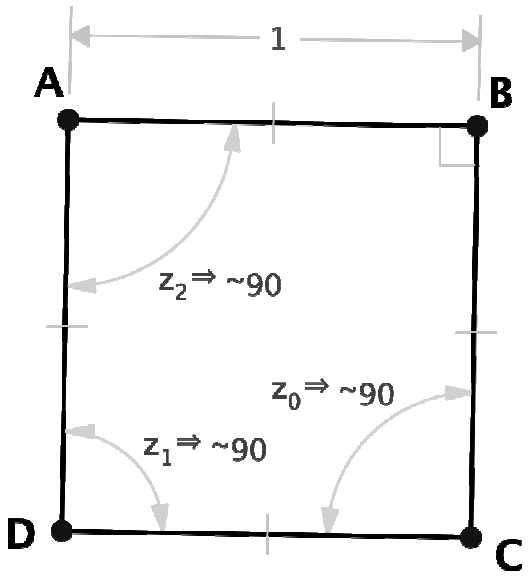
7. Because the sides are already constrained to be congruent, all the angles are now right. You can prove this, but GX can verify it!

GX can perform two types of measurements. It can give you a numeric answer, or it can calculate an algebraic expression for the measurement. Specify the type of measurement using the **CALCULATE** Toolbox. In this case, we want the actual measurement, so choose **REAL.**

Notice that the icons indicate which type of measurement is being used.

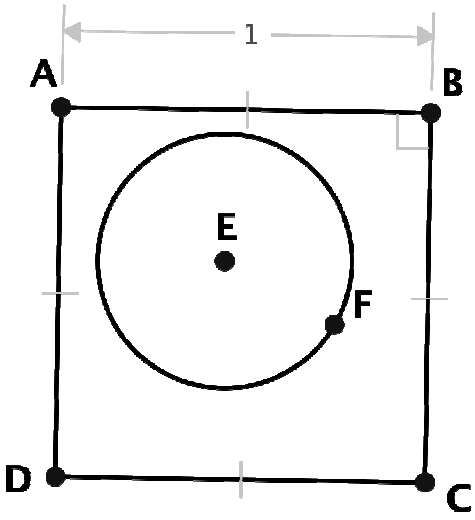


Choose two sides that form one of the other angles of the square and select **CALCULATE-ANGLE.** Try measuring the other angles for practice.



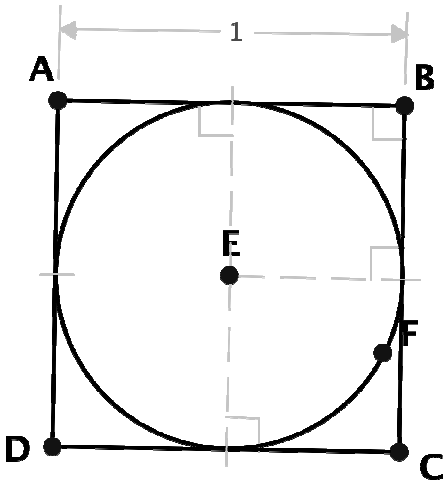
8. Try dragging and rotating the square by each of its endpoints and sides. You should notice that some objects in the drawing will move and others will not. It is important to understand the effects of the constraints that you place on each object. **Hide** or **DELETE** the angle measurements you made.

9. Choose to draw a circle in the square. Again, don't be particular about where you place the center and don't connect the circle to the square. GX will do that for us!



10. As explained in the introduction, we need the circle to touch each edge of the square. A **tangent line** is a line that touches the circle at exactly *one* point. So, we need to constrain the circle to be tangent to each side of the square.

Select the circle and one side of the square then choose to create the constraint.

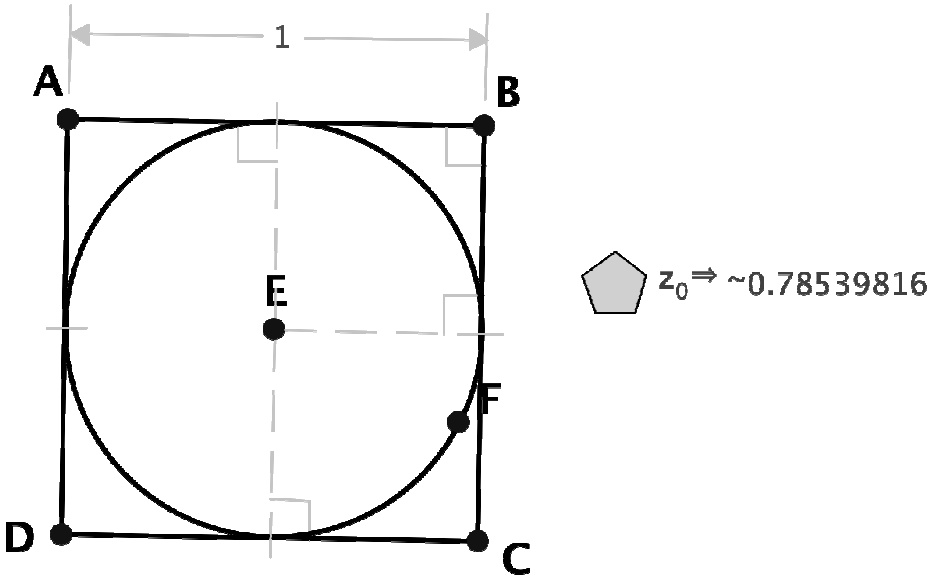


Repeat this process with two more of the sides. Because of the properties of the square, the remaining side will be tangent to the circle without constraining it manually.

Notice that GX shows the tangency by indicating that the radius of the circle at the point of tangency is perpendicular to the corresponding side of the square. This is an important property of circle tangents.

11. Finally, select the circle and find the calculate palette. Click on the real tab so the area is calculated as a real number and then choose to calculate the area. This icon is a bit misleading as a circle is certainly NOT a polygon!

12. You final model should look like this:



13. Have your teacher check your model.

Teacher Check:_____

Reporting Results

1. What percentage of the farmer's land can you tell him he will use? Express your answer to the nearest tenth of a percent.

2. We constructed the square by first constraining the sides to be equal. For practice, repeat the construction by creating a new document, but this time start by constraining the angles to be right angles.