Table of Contents

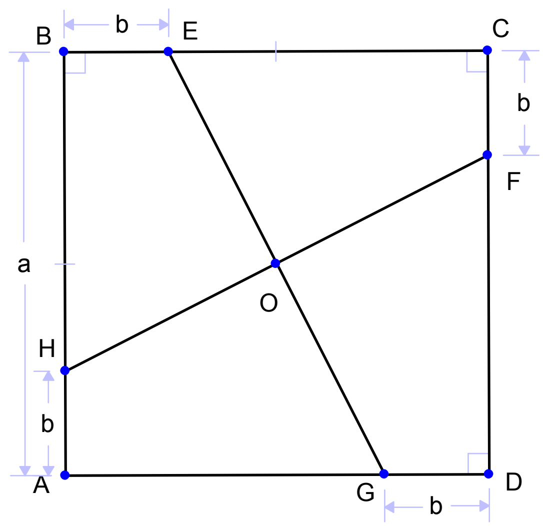
5. Segments in a Square – New Spin!

Problem Statement: In square ABCD, point E lies on BC, point F lies on CD, point G lies on DA, and point H lies on AB. Given that BE = CF = DG = AH, what is the relationship between segments EG and FH?

This problem is solved by transformations.

INVESTIGATION

1. Use **Toggle grid and axes** to display the axes without the grid.
2. Choose **Draw** → Line Segment and draw quadrilateral ABCD.
3. Constrain segments AB and BC to be perpendicular by selecting both segments and choosing **Constrain** → Perpendicular. Same way constrain BC ⊥ CD and CD⊥ AD.
4. Constrain AB = a by selecting the segment and choosing **Constrain** → Distance/Length.
5. Constrain BC = AB by selecting both segments and choosing **Constrain** → Congruent.
6. Draw point E on segment BC by choosing **Draw** → Point. Similarly, draw point F on CD, point G on DA, and point H on AB.
7. Constrain distance BE = b by selecting points B and E and choosing **Constrain** → Distance/Length. Similarly, constrain CF = DG = AH = b.
8. Draw EG and FH by choosing **Draw** → Segment.
9. Select segments EG and FH and choose **Construct** → Intersection. Label the point of intersection O.



Q1. What are the symmetries of a square? Which of those symmetries apply to the diagram above?

Q2. Are these transformations isometric (e.g. do they preserve distances and angles)?

10. Use *Geometry Expressions* to express the lengths EG and FH by selecting each segment and choosing **Calculate** → Distance/Length.
11. Use *Geometry Expressions* to calculate the angle measure between EG and FH.

Q3. Formulate your final conjecture.

PROOF

Q4. Prove your conjecture..

1. Delete all calculations.
2. Select square ABCD and all constructions by choosing **Edit** → Select All.
3. Choose **Construct** → Rotation, click on point O as center of rotation, and type 90.

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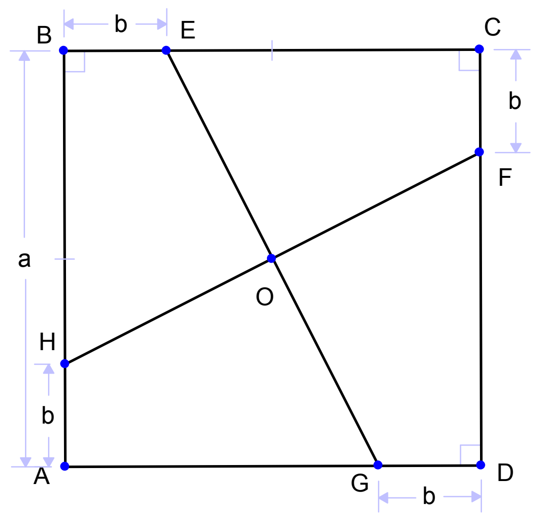


Q1. What are the symmetries of a square? Which of those symmetries apply to the diagram above?

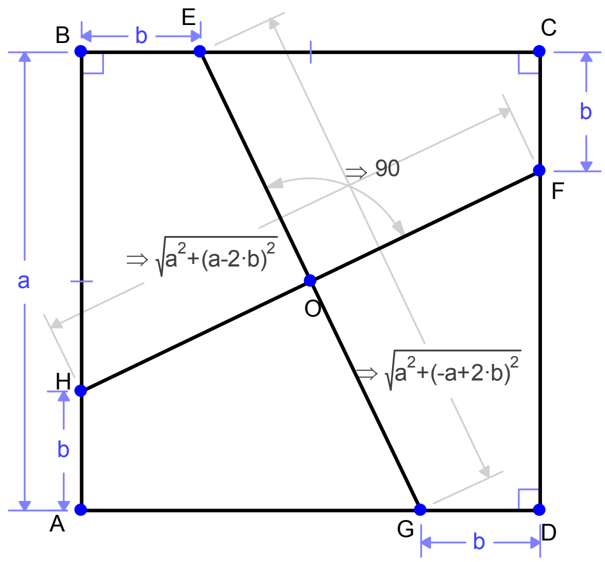
Students should list the four reflection symmetries (over the lines containing the diagonals and the lines containing the mid-segments), and the two rotational symmetries (clock- and counterclockwise by $90°$ and $180°$ about the center of the square). In our diagram, the rotational symmetries are the relevant ones.

Q2. Are these transformations isometric (e.g. do they preserve distances and angles)?

Yes, they are.

10. Use *Geometry Expressions* to express the lengths EG and FH by selecting each segment and choosing **Calculate** → Distance/Length.

11. Use *Geometry Expressions* to calculate the angle measure between EG and FH.



Q3. Formulate the final conjecture.

EG = FH, and EG $\perp$ FH.

PROOF

Q4. Prove your conjecture.

1. Delete all calculations.

2. Select square ABCD and all constructions by choosing **Edit** → Select All.

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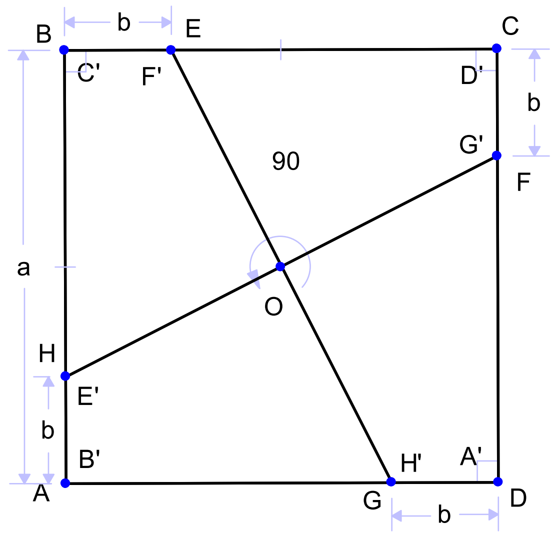

4. Due to the rotational symmatery of the square about its center, point C is rotated into point B and point D is rotated into point C. Therefore, CD $\rightarrow$ BC.

5. Now, point F $\in$ CD by construction. Due to rotation F $\rightarrow$ F′ such that F′ $\in$ BC.

6. Since rotation preserves distances, CF = BF′. By construction BE = CF, therefore F′ = E.

7. Similarly, E $\rightarrow$ H, H $\rightarrow$ G, and G $\rightarrow$ F. Thus, FH $\rightarrow$ EG so FH = EG and FH $\perp$ EG.