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Introduction

The National Council of Teachers of Mathematics (NCTM) Vision for School Mathematics invites us to "***Imagine a classroom, a school, or a school district where all students have access to high-quality, engaging mathematics instruction.***" **It goes on to describe how this may take place in the classroom:**

> ***Teachers help students make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures. Students are flexible and resourceful problem solvers. Alone or in groups and with access to technology, they work productively and reflectively, with the skilled guidance of their teachers. Orally and in writing, students communicate their ideas and results effectively. They value mathematics and engage actively in learning it.*** **(National Council of Teachers of Mathematics (NCTM).** ***Principals and Standards for School Mathematics.*** **Reston, VA: NCTM, 2000).**

Our goal in writing this book is to provide examples of how a symbolic geometry system, Geometry Expressions, can begin to make this happen. Geometry Expressions provides a playground where students can discover their own mathematics. They will begin to see mathematics as something that is created, not just a set of facts made up long ago. Once students take ownership of their mathematics, they will be more apt to "work productively and reflectively, with the skilled guidance of their teachers."

The graphical, interactive nature of Geometry Expressions brings life into a field that might otherwise seem irrelevant. The symbolics embedded in Geometry Expressions offer an algebraic view of the mathematics in concert with a geometric view, blurring the artificial line between the two. The smooth interface between Geometry Expressions and Computer Algebra Systems (CAS) adds another powerful resource for solving problems. These technologies can work together to change the way mathematics is done, in the same way that technology has changed the way architectural design is done; with computers managing the details while humans create the grand vision.

The units presented in this book are a jumping-off point for using Geometry Expressions in the classroom. Use the units to gauge the potential of this powerful software, and as a guide to applying Geometry Expressions in your own classroom. We trust that you will enjoy using the units and the software.

Lesson 1: Vertical Translations of Functions

Learning Objectives

This lesson gives a basic investigative approach to understanding the effect of adding a constant to a function.

Math Objectives

- The student will understand that the effect of adding a constant to a function is a vertical translation.
- The student will be able to determine the effect a change in the equation has on a graph, and vice verse with respect to vertical translations.
- The student will review the shapes of graphs and some key characteristics of the functions $y = x^2$, $y = \frac{1}{x}$, and $y = \sin(x)$

Technology Objectives

- The student will become proficient with the function feature of GX, and with using the variables tool to do dynamic investigations.

Math Prerequisites

- Students must know what a function is, and be familiar with three parent functions: $y = x^2$, $y = \frac{1}{x}$, and $y = \sin(x)$
- Students must be comfortable with radian measure for the sine function.
- Students should be familiar with function notation: y = f(x).

Technology Prerequisites

- Students should have a basic familiarity with Geometry Expressions. This can be accomplished with the “Intro to Unit Circle Trigonometry” lesson, or chapter 2 of the professional development materials.

Materials

- A computer with Geometry Expressions for each student or pair of students.
- Colored pencils (optional, but recommended).

Overview for the Teacher

Throughout this unit, we will be looking at function transformations through analysis of three families of functions, whose parent functions are $y = x^2$, $y = \frac{1}{x}$, and $y = \sin(x)$. For today's lesson, students will investigate vertical translations graphically, and analyze the correlation with symbolically adding a constant to a function. Vertical translations tend to be simple for students, and so the lesson also reviews some characteristics of each function family and helps students become more familiar with the software at the same time. With the use of the interactive software, students should be able to deduce the main ideas fairly easily on their own. The teacher's role today is mostly to facilitate and keep students on track. Students may need clarification and/or assurance that they understand the primary pattern correctly. You will particularly want to confirm student understanding of asymptotes and their equations on question 3, before they move on.

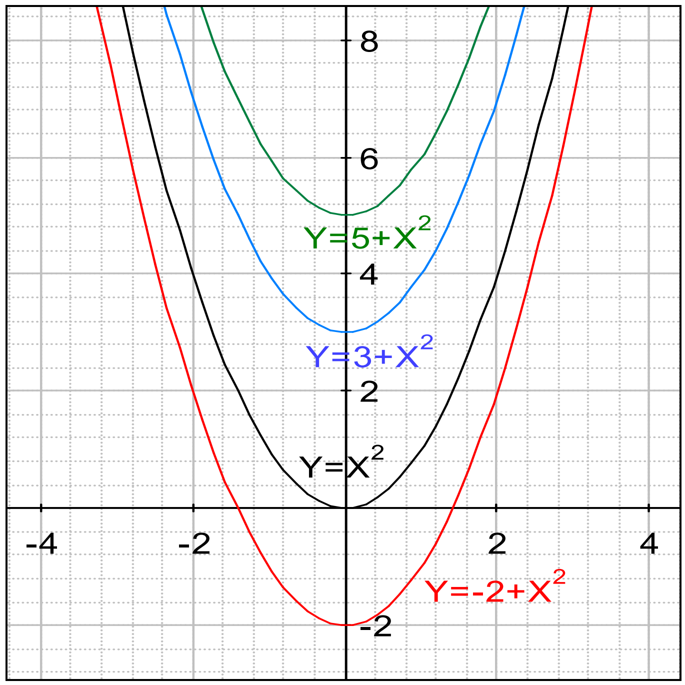
The second part should be done without the help of GX, so you may want to copy it on a different paper and hand it out as students leave the computers to work on their own. The software is a great tool for developing the ideas, but if we allow it to be overused, it can become a means for students to hide their lack of understanding. It is important that they can use the concepts they develop independently, and it is equally important that they know they are expected to learn the math, not just the computer application. However, the software can and will be used far more extensively later in the unit, for dynamic modeling, etc.

It will be helpful for students to have colored pencils and color-code their different transformations of the graphs. This is highly recommended both for ease of student understanding, and for ease of checking/grading.

For some classes, this lesson may get overly repetitive. Teachers should modify/shorten as appropriate for their particular group of students.

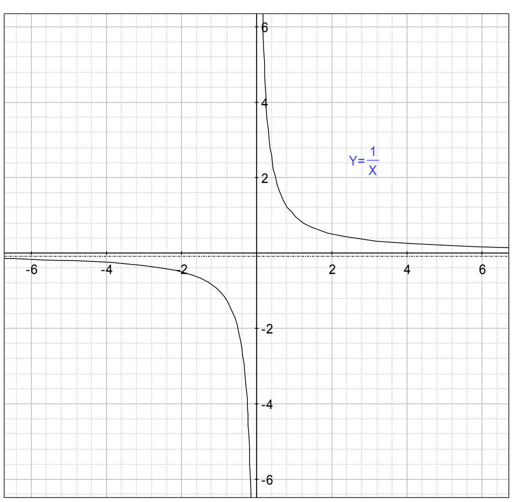
Vertical Translations part A

1) Teachers may want to introduce the animation tool at this point



2) Answers may vary slightly. The graph is moving up (vertical translation) by the number of units that is being added to the function. If it is a negative number, the graph is moving down.

3) A)



B) A vertical line through the origin;
A horizontal line through the origin.

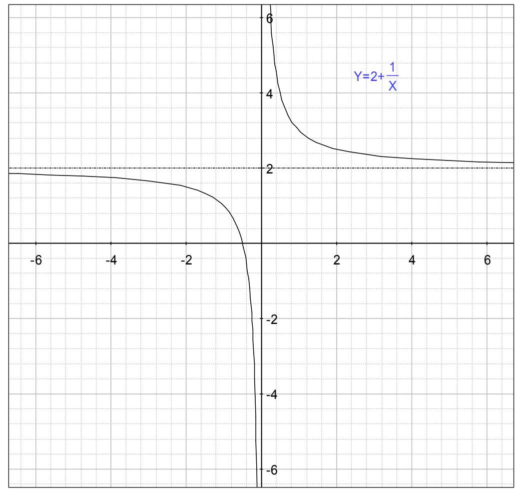
C) Y = 0; X = 0

It is important to check student understanding of asymptotes and their equations before they move on.

4)

A) Shifted the graph up 2 units (vertical translation).

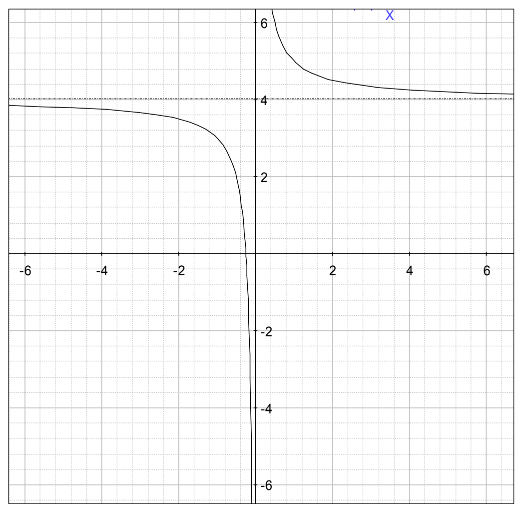
B) Y = 2
X = 0



5)

A) Shifted the graph up 4 units (vertical translation).

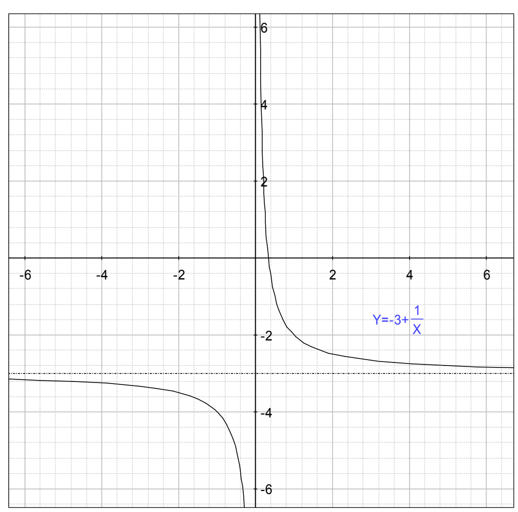
B) Y = 4
X = 0



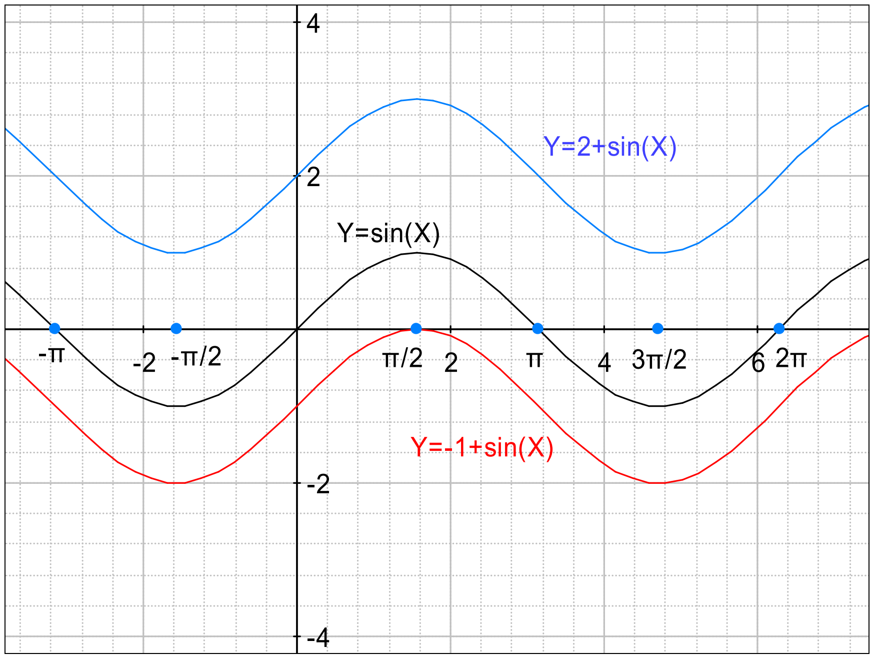
6)

A) Shifted the graph down 3 units (vertical translation).

B) Y = -3
X = 0



7)



8) The number added to a function shifts (translates) the function vertically by that many units. A positive number slides the graph up; a negative number slides the graph down.

Teacher Modeling: As students transition from the computer (possibly group) work to the independent practice, you may want to model the vertical translation of an arbitrary function on the board. A common issue is that students will draw a very rough, sloppy approximation of the general shape. Model the fact that they need to look at specific points and add the translation amount to each y-value. You may particularly want to model a curve, giving the example of identifying relative maximum and minimum values as reference points.